The Mathematics of Genius

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Experimentation in Mathematics: Plausible Reasoning in the 21st Century

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Mathematical genius appears to be a rare but remarkable phenomenon. Is it a genetic? An emergent accident of DNA? The result of good parenting? A stimulating intellectual environment? The product of inspiring teachers?

1. Recent interviews (Peter Liljedahl, SFU, unpublished PhD work) with top-level mathematicians suggests that most do not read very much, preferring to have results described to them, and put a large emphasis on the role of chance—which always favors the prepared mind.

2. Mathematical genius is almost always noted early in life and blossoms rather soon. As G. H. Hardy observed in his *A Mathematician's Apology*, "Mathematics, more than any other art or science, is a young man's game. ... I do not know an instance of a major mathematical advance initiated by a man past fifty. ... [Newton's] "greatest ideas of all, fluxions and the law of gravitation, came to him about 1666, when he was twenty-four." [3, pg. 78]. Needless to say, this does not bode well for the present authors!

3. Archimedes, Newton, Euler, Gauss, Ramanujan, and others all seem to have had extraordinary facilities for numerical and algebraic computation. Most of us know the story of Gauss who, when his teacher asked the class to sum the integers from 1 to 100, quickly noted that this was $50 \times 101 = 5050$, and was the only student to obtain the correct answer! Ramanujan's genius was his incredible skill at algebraic manipulation, a skill that flowered largely in a vacuum of advanced training in modern mathematics. The following is told of John von Neumann, who made fundamental contributions to computer science, mathematical economics, meteorology, probability theory, and quantum mechanics in the early 20th century [6, pg. 10]:

   > Two bicyclists are 20 miles apart and head toward each other at 10 miles per hour each. At the same time a fly traveling at a steady 15 miles per hour starts from the front wheel of the northbound bicycle. It lands on the front wheel of

> the southbound bicycle, and then instantly turns around and flies back, and after next landing instantly flies north again. Question: What total distance did the fly cover before it was crushed between the two front wheels?
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> The slow way of answering is to calculate the distance that the fly travels on its first trip to the southbound front wheel, then the distance it travels on its next trip to the northbound wheel, and finally to sum the infinite series so obtained. ... The short way is to note that the bicycles will meet exactly an hour after starting, by which time the 15-miles-per-hour fly must have covered 15 miles. When the question was put to [John von Neumann], he danced and answered immediately, "15 miles." "Oh, you've heard the trick before," said the disappointed questioner. "What trick?" asked the puzzled Johnny. "I simply summed the infinite series."

4. Others are known for their prodigious skill and boundless energy in ranging over a wide variety of very difficult topics with ease. The following is told of Nobel-prize-winning mathematical physicist Richard Feynman, during a lecture at CalTech [2, 5]:

   > Feynman immediately rose, astonishingly, to say that such objects would be gravitationally unstable. Furthermore, he said that the instability followed from general relativity. The claim required a calculation of the subtle countervailing effects of stellar forces and relativistic gravity. Fowler thought he was talking through his hat. A colleague later discovered that Feynman had done a hundred pages of work on the problem years before. The Chicago astrophysicist Subrahmanyan Chandrasekhar independently produced Feynman's result—it was part of the work for which he won a Nobel Prize twenty years later. Feynman himself never bothered to publish. Someone with a new idea always risked finding, as one colleague said, "that Feynman had signed the guest book and already left."

   John Maynard Keynes, who studied the original writings of Isaac Newton while riding taxicabs between British treasury board meetings,

wrote this of Newton on the tricentenary of his birth [4, 5]:

> His peculiar gift was the power of holding continuously in his mind a purely mental problem until he had seen straight through it. I fancy his pre-eminence is due to his muscles of intuition being the strongest and most enduring with which a man has ever been gifted. Anyone who has ever attempted pure scientific or philosophical thought knows how one can hold a problem momentarily in one's mind and apply all one's powers of concentration to piercing through it, and how it will dissolve and escape and you find that what you are surveying is a blank. I believe that Newton could hold a problem in his mind for hours and days and weeks until it surrendered to him its secret. Then being a supreme mathematical technician he could dress it up, how you will, for purposes of exposition, but it was his intuition which was pre-eminently extraordinary—"so happy in his conjectures," said de Morgan, "as to seem to know more than he could possibly have any means of proving."

5. In a recent provocative essay, David Lykken explores the emergence of genius. Here is his conclusion [5]:

> Ericsson and Charness [1] are willing to acknowledge that genetic differences in temperament and "preferred activity level" may determine which of us go for the gold but, curiously, they cling to the assumption that individual genetic differences in both physical and mental capacities are not important, perhaps nonexistent. This would require us to believe that ... little Gauss's ability to correct his father's arithmetic at three and confound his school master at ten resulted, not from extraordinary mental hardware, but from mental software acquired through self-directed practice in an intellectually unstimulating environment.
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> Those of us who have studied MZ [monozygote, i.e., identical] twins reared apart from one another find these assumptions ... incredible. We cannot believe that MZA twins [monozygote twins reared apart] correlate .75 in IQ merely because,

in their separate environments, their similarities in temperament led them to indulge in very similar amounts of practice on very similar topics. ...

I think we must agree with Ericsson, however, that works of genius tend to be the product of minds enriched by years of concentrated effort. Isaac Newton often became so caught up in cerebration that he would forget to eat or sleep. Edwin Land, inventor of the instant Polaroid camera and of a sophisticated computational theory of color vision, sometimes worked at his desk for 36 hours or more, unaware of the passage of time until he felt faint on standing up. Similar stories were told of Edison. It does not follow, however, that these were ordinary minds to begin with.

Edison, Feynman, Land, and Newton all from their boyhood had intense curiosity, an enthusiasm or zeal for discovery and understanding. Each of them was able to take seriously hypotheses that others thought to be implausible (or had not thought about at all). All four possessed a kind of intellectual arrogance that permitted them to essay prodigious tasks, to undertake to solve problems that most of their contemporaries believed to be impossible. And each of them had quite extraordinary powers of concentration. ...

I think what lies at the heart of these mysteries is genetic, probably emergenic. The configuration of traits of intellect, mental energy, and temperament with which, during the plague years of 1665–6, Isaac Newton revolutionized the world of science were, I believe, the consequence of a genetic lottery that occurred about nine months prior to his birth, on Christmas day, in 1642.

References

[1] K. Anders Ericsson and Neil Charness. Expert Performance: Its Structure and Aquisition. *American Psychologist*, 49:725–747, 1994.

[2] James Gleick. *Genius: The Life and Science of Richard Feynman.* Pantheon Books, New York, 1992.

[3] Godfrey H. Hardy. *A Mathematician's Apology.* Oxford University Press, Oxford, 1971.

[4] John M. Keynes. Newton the Man. In James R. Newman, editor, *The World of Mathematics*, volume I. Simon and Schuster, New York, 1956.

[5] David T. Lykken. The Genetics of Genius. In Andrew Steptoe, editor, *Genius and the Mind: Studies of Creativity and Temperament.* Oxford University Press, Oxford, 1998.

[6] Norman MacRae. *John von Neumann: The Scientific Genius Who Pioneered the Modern Computer, Game theory, Nuclear Deterrence, and Much More.* American Mathematical Society, Providence, 1992.